SHA-3 Proposal: FSB

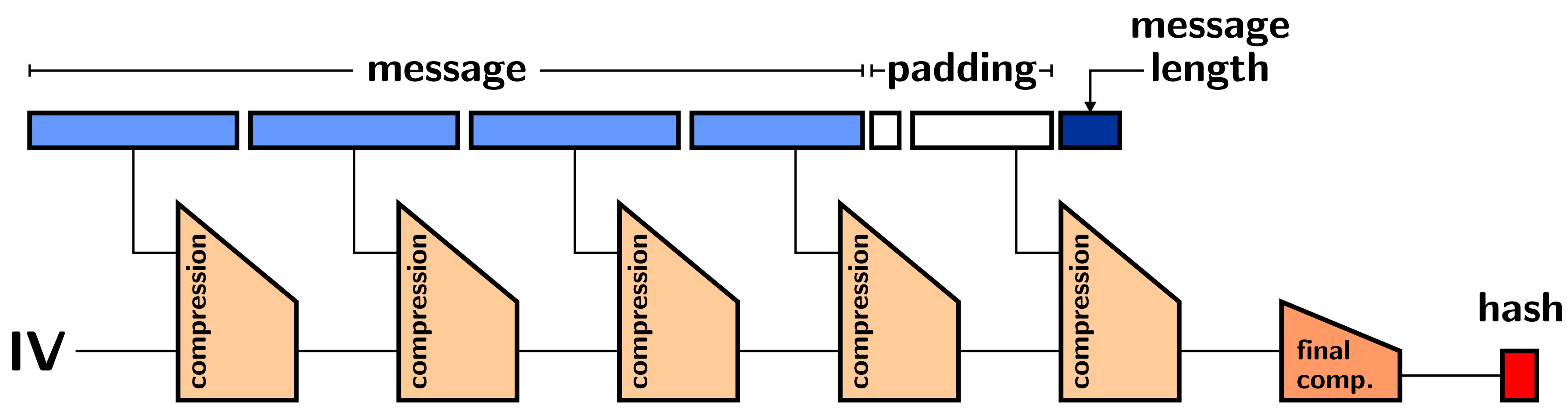
D. Augot, M. Finiasz, P. Gaborit,
S. Manuel, and N. Sendrier

INRIA

ENSTA ParisTech

High Overview of FSB

- FSB uses the Merkle-Damgård construction (chaining and padding), with a large internal state:
  → it uses a final compression function.
- the main compression function uses a one-way function from coding theory:
  → security reduction for inversion and collision search.

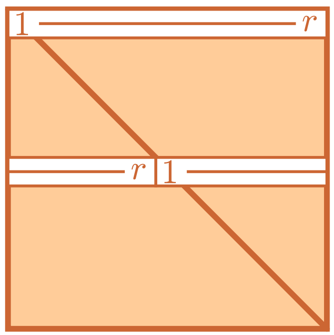


- The compression function of FSB is made of two steps:
  - a non-linear bijective step,
  - a linear compression step.

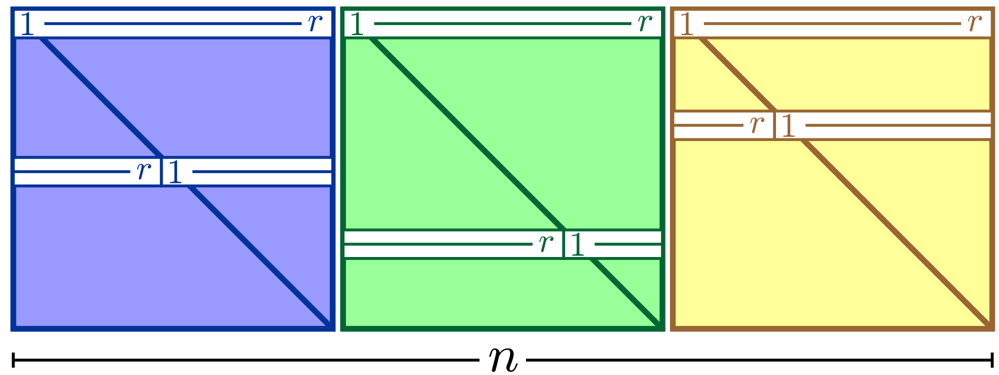
- First the s input bits are transformed in a binary vector of length n and Hamming weight w:
  - for efficiency we use regular words.
- Then this vector is multiplied by a binary matrix $\mathcal{H}$
  - $w \ll n$ so this is simply the XOR of w columns of $\mathcal{H}$.

- In practice $\mathcal{H}$ is a truncated quasi-cyclic matrix

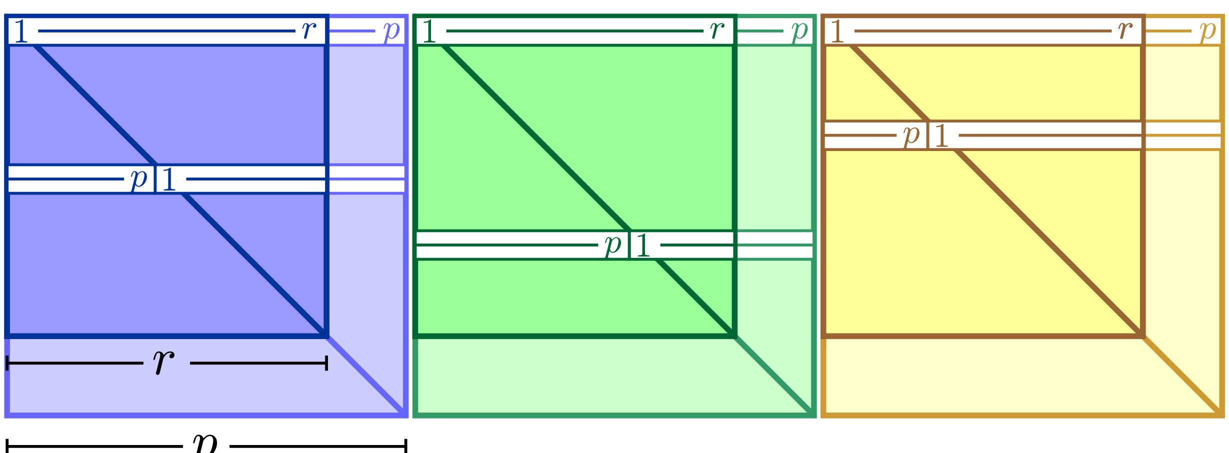
circulant



quasi-cyclic



truncated quasi-cyclic



- In practice $\mathcal{H}$ is a truncated quasi-cyclic matrix
  - $\mathcal{H}$ is described by its first line: $\frac{n}{r}$ vectors of p bits.
  - columns of $\mathcal{H}$ are truncated cyclic shifts of these binary vectors.
  - which vectors to choose and how much they should be shifted depends on the input:
    - w indexes are derived from 13 or 14 input bits each,
    - 8 IV/chaining bits and 5 or 6 message bits,
    - the i-th index is taken in the interval $\left[i\frac{n}{w}, (i+1)\frac{n}{w}-1\right]$,
    - the w indexes correspond to the w columns to XOR.

The best algorithms that can be used to attack FSB are:

- Generalized birthday algorithm
  - best algorithm for inversion and second preimage,
  - requires a lot of memory.
- Information set decoding
  - best algorithm for collision search,
  - yields strong constraints on the choice of r and w.

- Proposed parameters have been chosen according to these algorithms, plus a security margin.

- Inverting the compression function requires to find w columns of $\mathcal{H}$ which XOR to a target vector.
  - ▷ this is an instance of the syndrome decoding problem,
  - ▷ this problem is NP-complete for random matrices, but also for truncated quasi-cyclic matrices,
  - ▷ well chosen values of p and r give supposedly hard instances of the problem.

- Collisions require $2w$ columns of $\mathcal{H}$ which XOR to 0.
  - ▷ also an instance of the syndrome decoding problem,
  - ▷ an "easier" instance in practice.

- An important point is that these reductions are tight.

adversary	best attack	reduction
collision	ISD$(n, r, 2w) \times 1$	CSD$(n, r, 2w)/1$
preimage	GBA$(n, r, w) \times 1$	CSD$(n, r, w)/1$
second-preimage	GBA$(n - w, r, w) \times 1$	CSD$(n - w, r - w, w)/1$

ISD $=$ Information set decoding
GBA $=$ Generalized birthday algorithm
CSD $=$ Computationnal syndrome decoding.

- One call to the adversary solves the CSD problem, one call to ISD/GBA is enough to build an adversary.

Final Compression Function

Few constraints apply to the final compression function.

- it must not weaken the main compression function
  - ▷ any linear function is bad
    $\longrightarrow$ simple truncation is impossible.
- it does not require collision resistance/one-wayness
  - ▷ collisions on the final compression do not directly lead to collisions on FSB
- Cryptographers and the NIST need to be convinced...
  - ▷ anything too simple should be avoided.

We propose to use Whirlpool [Rijmen, Barreto 2004]:

- The r-bit output of the main compression function is input as an r-bit message to Whirlpool
  - ▷ the final output is a truncated Whirlpool hash.

This is a safe choice, not an efficiency oriented choice:

- ▷ Whirlpool is highly non-linear,
- ▷ we are confident that it is a secure hash function,
- ▷ attacks on Whirlpool would probably not affect our construction.

- The main compression functions is very simple:
  - shift and XOR w times some vectors
    - $\longrightarrow$ with precomputed shifts, only XORs are required.
  - parameters of FSB are quite large
    - $\longrightarrow$ the XORs are expensive: 250 to 500 cycles/byte.

- The description of FSB is large:
  - 2 millions bits from digits of π define the vectors
    - $\longrightarrow$ this is a problem for constrained environments,
  - using pseudo-random data could improve this but would loosen the security reduction.

Conclusion

- The main interest of FSB is its compression function:
  - inversion and collision search reduce to hard problems,
  - it is slow, but much faster than most “similar designs,”
  - it is very simple to describe/implement
    $\longrightarrow$ only very basic operations are used,
  - the description of FSB is large as “random bits” are needed.

- Security reduction to hard problems comes at a cost, but it can be practical in many contexts.